MATERIAL AND ENERGY BALANCES ON A RUNNER

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INTRODUCTION

Exercise physiologists have traditionally used direct calorimetry and/or respiratory gas analysis to evaluate the energy consumption of various physical activities. This paper presents an alternative to estimating the caloric cost using the universality of thermodynamics coupled with basic chemical engineering principles. The procedure can be applied to any physical activity. For demonstration, running is used. The analysis does not require a laboratory setting but rather uses the runner's natural environment outdoors.

I. MACROSCOPIC ENERGY BALANCE

The macroscopic energy balance is a statement of the law of conservation of energy. In the most general sense, the energy accumulation within a system is the difference between all entering energy and all exiting energy (for nonnuclear processes). This is the first law of thermodynamics. A more specific statement of this principle segregates the modes of energy into those associated with material flowing across the boundary, heat, and work. It is

$$\left\{\begin{array}{l}\text{Rate of accumulation}\\ \text{of internal, kinetic,}\\ \text{and potential energy}\end{array}\right\} = \left\{\begin{array}{l}\text{Rate of internal,}\\ \text{kinetic, potential}\\ \text{energy in by}\\ \text{convection}\end{array}\right\} - \left\{\begin{array}{l}\text{Rate of internal,}\\ \text{kinetic, potential}\\ \text{energy out by}\\ \text{convection}\end{array}\right\}$$

$$+ \left\{\begin{array}{l}\text{Net rate of heat}\\ \text{to system from}\\ \text{surroundings}\end{array}\right\} - \left\{\begin{array}{l}\text{Net rate of work done}\\ \text{by system on surroundings}\end{array}\right\}$$

The following algebraic equation mathematically represents this statement:

$$d[M_{SYS}(\hat{U} + \hat{K} + \hat{P})_{SYS}] = \left(\sum_{i=1}^{n} (\hat{H}_i + \hat{K}_i + \hat{P}_i)dm_i\right)_{IN} - \left(\sum_{j=1}^{m} (\hat{H}_j + \hat{K}_j + \hat{P}_j)dm_j\right)_{OUT} + dQ - dW \quad (1)$$

Where:
M_{SYS} = Mass of system
$\hat{U}_{SYS}$ = Specific internal energy of the system
$\hat{K}_{SYS}$ = Specific kinetic energy of the system
$\hat{P}_{SYS}$ = Specific potential energy of system
$\hat{H}_i$ = Specific enthalpy of entering material "i"
$\hat{K}_i$ = Specific kinetic energy of entering material "i"
$\hat{P}_i$ = Specific potential energy of entering material "i"
m_i = Mass of entering material "i"
$\hat{H}_j$ = Specific enthalpy of exiting material "j"
$\hat{K}_j$ = Specific kinetic energy of exiting material "j"
$\hat{P}_j$ = Specific potential energy of exiting material "j"
m_j = Mass of exiting material "j"
Q = Heat transferred across the system boundary
W = Work done by the system on the surroundings

Equation (1) applies to a thermodynamic system, a specific region of space which is under consideration, separated from its surroundings by a boundary. Note that the left side of the equation refers only to energy changes within the system and is composed of state functions whereas the right side of the equation refers to energy transfers across the system boundaries and depend on the path followed. The sign convention adopted for Q and W is consistent with that used by most thermodynamicists. Q is taken to be positive when heat is flowing into the system from the surroundings while W is taken to be positive when the system does work on the surroundings.

For a rigorous derivation of the macroscopic energy balance and precise definitions of thermodynamic terms, the reader is referred to Introduction to Chemical Engineering Thermodynamics by J. M. Smith and H. C. VanNess.

II. APPLICATION TO A RUNNER

In order to apply the macroscopic energy balance, we must select a system. We choose the system to be the runner, his shoes, his clothing, and all objects he wears continuously throughout his run. The boundary is the outermost surface of the system.

The evaluation of thermodynamic state functions requires knowledge of the initial state, the final state, and a reference state. The runner starts and finishes his run with the same velocity at the same elevation so that changes in kinetic energy and potential energy, respectively, of the system are zero.

The runner loses energy by heating and humidifying respiratory gases, by convective heat transfer from his surface to the air, by evaporative heat losses through the sweating mechanism, by conduction through the soles of his shoes to the ground, and by doing work to overcome the trangential force of the surrounding air medium. The runner gains energy by intercepting radiant energy from the surroundings. The interaction of these energy gains and losses is balanced by the metabolic generation of heat which ultimately increases the heat content of the body. Therefore, the integrated form of the macroscopic energy balance is

$$(M_f\hat{U}_f - M_c\hat{U}_o) = (\hat{H}_{O_2}m_{O_2} + \hat{H}_{H_2O}m_{H_2O} + \hat{H}_{N_2}m_{N_2})_{IN} - (\hat{H}_{O_2}m_{O_2} + \hat{H}_{CO_2}m_{CO_2} + \hat{H}_{N_2}m_{N_2} + \hat{H}_{H_2O}m_{H_2O})_{OUT} - (\hat{H}_{fg}m_{H_2O})_{EVAP} + Q_{RAD} + Q_{CONV} + Q_{COND} - W_{WIND} \quad (2)$$

Where: M_f = Final mass of runner

$\hat{U}_f$ = Final specific internal energy of runner

M_o = Initial mass of runner

$\hat{U}_o$ = Initial specific internal energy of runner

$\hat{H}_{fg}$ = Specific enthalpy of vaporization of water

$M_{H_2O_{EVAP}}$ = Mass of water evaporated from surface

Q_{RAD} = Radiant heat absorbed by runner

Q_{CONV} = Convective heat transfer to runner

Q_{COND} = Conductive heat transfer to runner

W_{WIND} = Work done by runner against wind

and the first two terms on the right side of equation (2) arise because of respiratory gas exchange. The enthalpies of these gaseous compounds are much greater than their kinetic and potential energy changes allowing us to neglect the latter two.

Note that equation (2) contains no work terms arising from acceleration changes or center of mass elevation changes. We restrict our definition of work to include only energy in transition across the system boundaries by virtue of an energy potential other than temperature. Energy transfers occurring wholly within a system are not considered as work. Additional work terms do arise, however, when the runner runs in sand, in loose gravel, or on soft ground.

The following definitions are made to facilitate writing equation (2):

1) $\Delta(M\hat{U})_{SYS} = (M_f\hat{U}_f - M_o\hat{U}_o)_{SYS}$

2) $$\Delta H_{LUNG} = (\hat{H}_{CO_2}m_{CO_2} + \hat{H}_{H_2O}m_{H_2O} + \hat{H}_{O_2}m_{O_2} + \hat{H}_{N_2}m_{N_2})_{OUT} - (\hat{H}_{H_2O}m_{H_2O} + \hat{H}_{O_2}m_{O_2} + \hat{H}_{N_2}m_{N_2})_{IN}$$

3) $\Delta H_{EVAP} = \hat{H}_{fg} m_{H_2O}$

Introducing these, equation (2) becomes

$$\Delta(M\hat{U})_{SYS} = -\Delta H_{LUNG} + \Delta H_{EVAP} + Q_{RAD} + Q_{CONV} + Q_{COND} - W_{WIND} \quad (3)$$

The runner carries his fuel with him and combusts it as it is needed. The left side of equation (3) incorporates the heat produced through chemical reaction as well as sensible heating due to the change in the average system temperature. This is dealt with in the following way:

$$\Delta(M\hat{U})_{SYS} = \Delta(M\hat{U})_{SENS} + \Delta U_{iG}$$

Where: $\Delta(M\hat{U})_{SENS}$ = Change in internal energy due to heat effects

ΔU_{iG} = Change in internal energy due to internal generation of heat (metabolism)

The metabolic generation of heat is given by

$$\Delta U_{iG} = -\Delta H_{LUNG} - \Delta H_{EVAP} + Q_{RAD} = Q_{CONV} - W_{WIND} + Q_{COND} - \Delta(M\hat{U})_{SENS} \quad (4)$$

This equation is the working model of the macroscopic energy balance as applied to a runner. It provides a means of calculating the caloric expenditure of running.

III. EVALUATION PROCEDURE FOR EACH TERM

The calculational procedures for the terms comprising equation (4) are outlined in the following paragraphs.

The change in internal energy due to heat effects is given by

$$\Delta(M\hat{U})_{SENS} = MC_v\,(\theta_f - \theta_o) \quad (5)$$

where: M = average mass of runner

C_v = constant volume heat capacity of human tissue

θ_f = final average system temperature

θ_o = initial average system temperature

The temperature distribution within the body is assumed to qualitatively resemble that shown in Figure (1). Consequently, the average system temperature change is approximated as

$$\theta_f - \theta_o = 0.7(T_{R_f} - T_{R_o}) \tag{6}$$

where: T_R = Rectal temperature

The specific heat of most tissue is about 0.83. Obviously, the change in sensible heat content in the body is calculated inexactly. Experimental evidence indicates that these imprecisions are of no consequence when weighed against the magnitudes of other terms of equation (4).

Heat is transferred to the surroundings through respiratory gases in two ways - sensible heating of ambient air and vaporization of water to water vapor in the lungs. To facilitate the calculation of the enthalpy change, we choose the reference state to be pure components at a temperature of 30°C and a pressure of 101.33 kPa, a close approximation of the temperature and pressure of exhaled gases. This choice eliminates the carbon dioxide enthalpy term. The remaining equation becomes

$$\Delta H_{LUNG} = (\hat{H}_{O_2} m_{O_2} + \hat{H}_{N_2} m_{N_2} + \hat{H}_{H_2O} m_{H_2O})_{OUT} - (\hat{H}_{O_2} m_{O_2} + \hat{H}_{N_2} m_{N_2} + \hat{H}_{H_2O} m_{H_2O})_{IN} \tag{7}$$

Equation (7) is simplified to

$$\Delta H_{LUNG} = m_g C_p\ (30° - T_a) + m_{H_2O_L} \lambda \tag{8}$$

where: m_g = Mass of respiratory gases breathed during run

C_p = Average constant pressure heat capacity of respiratory gases

30°C = Assumed exit temperature of gases

T_a = Average ambient temperature of inhaled gases

$m_{H_2O_L}$ = Mass of water vaporized in lungs

λ = Heat of vaporization of water at exit temperature of gases

The heat capacity of respiratory gases is taken to be that of air at 25°C, $1.005^{KJ}/(kg\cdot k)$. The mass of respiratory gases breathed during the run is calculated from

$$m_g = V^\circ n \rho t \tag{9}$$

where: v° = Tidal volume; volume per breath

n = Breathing frequency

ρ = Density of air

t = Duration of run

Equation (9) assumes that a constant breathing pattern is developed and maintained. The mass of water vapor saturating the inhaled air is given by

$$M_{H_2O_L} = m_g\ (\mathfrak{H}_{LUNG} - \mathfrak{H}_a) \tag{10}$$

where: $\mathfrak{H}_{LUNG}$ = humidity of lung; saturation humidity at 30°C; kgH_2O/kg dry air

$\mathfrak{H}_a$ = ambient humidity; kgH_2O/kg dry air

The ambient humidity is determined using a sling psychrometer and psychrometric chart. Sensible heating of respiratory gases is a small term, corresponding to approximately two percent of metabolic energy production during winter months. Saturation of air with water vapor during respiration amounts to 10 to 15 percent of metabolic energy production.

The evaporation of water from the surface of the body is a term of large magnitude, equal to approximately 40 to 70 percent of metabolic energy production, depending upon the seasons. It is calculated using

$$\Delta H_{EVAP} = m_{H_2O_s}\lambda_s + m_{H_2O_s}(\hat{H}_s - \hat{H}_{30°C}) \tag{11}$$

where: $m_{H_2O_s}$ = Mass of water evaporated from the runner's surface

λ_s = Latent heat of vaporization of water at surface temperature

$\hat{H}_s$ = Specific enthalpy of saturated liquid water at surface temperature

$\hat{H}_{30°C}$ = Specific enthalpy of saturated liquid water at 30°C

The mass of surface-evaporated water is determined from

$$m_{H_2O_s} = m_t - m_{H_2O_L} - m_R \tag{12}$$

where: m_t = Net mass lost by runner (from scale weighings)

$m_{H_2O_L}$ = Mass of water lost by vaporization in lungs

m_R = Mass loss due to combustion of stored fuels

The mass of carbohydrate and fat metabolized during the run constitutes m_R. It is approximated by

$$m_R = 0.0133d \tag{13}$$

where m_R has the units of kilograms and d is the length of the run in miles.

The runner loses energy by doing work to overcome the force of the air on his projected area, since a body immersed in a fluid experiences form drag. From fluid mechanics, there are correlations for drag coefficients as functions of relative fluid velocity. (see figure (2).) In using these, it is assumed that the geometric shape of a runner most closely approximates that of a cylinder perpendicularly oriented relative to the direction of travel of the fluid stream. Work against wind as a function of net wind velocity is plotted in figure (3). The direction of the fluid stream must be considered in assessing the work term. Since the amount of work done depends upon the running course, a suitable average must be used. This is not critical since the magnitude of this work term is rarely greater than five percent of metabolic production of heat.

Conduction of heat from the feet through the soles of the shoes to the ground is very difficult to measure and probably of no significance as a result, Q_{COND} is neglected in the calculations.

The general equation governing the amount of heat lost to a fluid by convection from a solid surface at a higher temperature, known as Newton's law of cooling, is

$$Q_{CONV} = hA\,(T_a - T_s)t \qquad (14)$$

where:
- Q_{CONV} = Amount of heat transferred by convection
- h = Heat transfer coefficient
- A = Area over which heat transfer occurs
- T_a = Ambient fluid temperature
- T_s = Solid surface temperature
- t = Duration of convective heat transfer

Since the body has areas of various surface temperatures, the total convective heat transfer is the sum of each of the area-specific convective heat transfers for both the dry surface before onset of sweating and the wet surface after perspiration has begun. Therefore, equation (14) becomes

$$Q_{CONV} = \sum_{i=1}^{2} \sum_{j=1}^{n} h_{ij}\, A_j\, (T_a - T_{ij})t_i \qquad (15)$$

where summation with respect to "i" accounts for the dry and wet periods and summation with respect to "j" indicates that different areas of the body will exhibit different surface temperatures. Inherent difficulties in determining area-specific heat transfer coefficients require using an overall heat transfer coefficient so that equation (15) becomes

$$Q_{CONV} = h \sum_{i=1}^{2} \sum_{j=1}^{n} A_j (T_a - T_{ij}) t_i \tag{16}$$

The heat transfer coefficient, h, is determined from the mass transfer coefficient, k', by using the Chilton-Colburn relation. This relation provides an analogy between heat transfer, mass transfer, and momentum transfer occurring simultaneously. The correlation which equates dimensionless heat transfer groups with dimensionless mass transfer groups, with appropriate simplifications, is

$$\frac{h}{k'} = C_{p_f} \left(\frac{S_c}{P_r} \right)_f^{0.56} \tag{17}$$

where:
- S_c = Schmidt number = $\mu/\rho D_{AB}$
- P_r = Prandtl number = $C_p\mu/k$
- C_{pf} = Constant pressure heat capacity of fluid medium
- μ = viscosity of fluid
- ρ = Density of Fluid
- D_{AB} = Binary diffusivity of water vapor and air
- k = Thermal conductivity of fluid

and "f" = Refers to the properties of the fluid.

The mass transfer coefficient, k', is obtained by applying the general equation:

$$(\text{mass transfer coefficient}) = - \frac{(\text{mass flux})}{(\text{driving force})} \tag{18}$$

Specifically, Equation (18) becomes

$$k' = \frac{\left(m_{H_2O_s} / A_s t_w \right)}{(\mathcal{H}_s - \mathcal{H}_a)} \tag{19}$$

where: $m_{H_2O_s}$ = Mass of water evaporated from the runner's surface (from equation 12)

$\mathcal{H}_s$ = Saturation humidity at runner's average wet surface temperature, kgH_2O/kg dry air

$\mathcal{H}_a$ = Ambient Air humidity, kgH_2O/kg dry air

t_w = Duration of run with wet surfaces

A_s = Surface area over which evaporation is occurring.

Using this equation requires knowledge of body surface temperatures. A good surface area-average can be obtained by using thermistors strapped to key areas of the body. These temperatures vary with time into the run so that a time-average must be estimated for each area. Noting the wind velocity on each of these outings enables one to develop an experimental correlation for mass transfer coefficient as a function of wind velocity (as measured by an anemometer). Once developed, this correlation negates the necessity of recording body surface temperatures. When the mass transfer coefficient has been determined, the heat transfer coefficient is calculated using equation (17). If surface temperatures are measured, equation (16) is used to compute the convection term. If they are not directly measured, the average surface temperature is indirectly obtained using equation (19) and a psychrometric chart. The convection term is then calculated by equation (14).

The importance of the convective heat transfer term varies from insignificance in hot, humid, stagnant environments to importance in cold, dry, windy environments.

The determination of the radiant heat exchange of the runner with his surroundings cannot easily be calculated directly. It is assessed experimentally as described in the next section.

IV. DETERMINATION OF RADIANT FLUX

In order to determine the radiant flux, a parallel experiment is performed while the runner is running.

The radiant flux is assessed by applying the macroscopic energy balance (equation (1)) to a stationary cloth-covered hemispherically shaped metal bowl. The cloth covering is just saturated with water at the start of the experiment. The bowl is set outdoors facing convex upward on a good insulator to minimize heat transfer by conduction to the ground. Heat is transferred between the bowl and its surroundings in the form of convection, evaporation, and radiation. The simplified energy balance is given by

$$Q_R = \Delta U + \Delta H_{EVAP} - Q_{CONV} \tag{20}$$

where: Q_R = Radiant energy absorbed by the bowl

ΔU = Change in heat content of the bowl

ΔH_{EVAP} = Energy transfer due to evaporation of water

Q_{CONV} = Heat transfer by convection

The terms on the right side of equation (20) are given by

$$\Delta U = M_B C_V (T_f - T_i) \tag{21}$$

$$\Delta H_{EVAP} = m_{H_2O}\lambda \tag{22}$$

$$Q_{CONV} = hA_B(T_a - T_s)t \tag{23}$$

where: M_B = Average mass of bowl during experiment

C_V = Constant volume heat capacity of bowl

T_f = Final average surface temperature of bowl

T_i = Initial average surface temperature of bowl

m_{H_2O} = Mass of water evaporated during experiment

λ = Latent heat of vaporization of water at average bowl temperature T_s

h = Heat transfer coefficient

A_B = Surface area of bowl

T_a = Average ambient air temperature

T_s = Average bowl surface temperature for the duration of the experiment

t = Duration of the experiment

The mass of evaporated water is obtained from initial and final scale readings. A sufficient quantity of surface temperature readings are made in order to make a reliable estimate of the average surface temperature, both initially and finally. The ambient temperature, both wet bulb and dry bulb, are recorded at these times also. The initial and final bowl humidities and initial and final air humidities are found using a psychrometric chart. Then, the heat transfer coefficient is obtained from the mass transfer coefficient via equations (19) and (17) as was previously done.

The total radiant energy exchange is calculated using equation (20), but then must be expressed as a flux, rate per unit area, to be used in the energy balance for the runner. The radiant flux, R, is given by

$$R = Q_R/A_R t \tag{24}$$

where: A_R = Surface area of bowl which is absorbing radiation

t = Duration of experiment

Note that A_R varies between projected area perpendicular to incident beam and total surface area for clear days and overcast days, respectively.

On clear days,

$$A_R = A_A\cos\theta + A_s\sin\theta \tag{25}$$

where: A_A = Projected bowl area from vertical view

A_s = Projected bowl area from horizontal view

θ = Angle between incident beam and vertical

The angle θ is found by measuring the shadow length of an object of known height,

$$\theta = \arctan \frac{\text{(shadow length)}}{\text{(height)}}$$

on overcast days, radiation is scattered and diffused by the cloud cover so that no distinct shadows appear. Under these circumstances,

$$A_R = A_B$$

where A_B is the total surface area of the bowl.

The radiant flux varies from near zero to nearly $0.800^{kW}/m^2$ on heavy overcast and very clear days, respectively.

The radiant energy transferred to the runner is given by

$$Q_{RAD} = RATf \tag{26}$$

where: R = Radiant flux

A = Runner's surface area receiving radiation

t = time of run

f = Fraction of time t runner is exposed to radiation; correction for shade

In the absence of direct solar radiation, the area over which radiation transfer occurs is the runner's entire surface area. For those days on which distinct shadows are present, the projected area perpendicular to the solar incident beam must be used. This area is calculated using the dimensions of the runner's shadow on the ground:

$$A = \frac{\ell_1+\ell_2}{2} \quad w\,(\cos\theta)_{AV}$$

$$(\cos\theta)_{AV} = \frac{1}{2}\left((\cos\theta)_1 + (\cos\theta)_2\right)$$

$$(\cos\theta)_1 = \cos\left[\arctan\left(\frac{\ell_1}{h}\right)\right]$$

$$(\cos\theta)_2 = \cos\left[\arctan\left(\frac{\ell_2}{h}\right)\right]$$

where

ℓ_1 = Length of shadow at start of run

ℓ_2 = Length of shadow at end of run

h = Height of runner

w = Average shadow width of runner

The factor f is a correction allowing for the fact that the runner may not be exposed to the radiant flux R consistently throughout his run. (i.e. he may run in the shade af trees and buildings.) The fraction of exposure is subjectively estimated.

V. CONCLUSION

With the results of the last section, the right side of equation (4) is completely specified and therefore, the energy expenditure of running can be determined. The analysis described here can readily be extended to other activities as well.

The analysis says nothing of the magnitude or consistency of the metabolic generation term. All it says is that the energy must be balanced. The next step is to perform longitudinal studies on various subjects to determine the impact of environmental parameters on metabolic generation.

The dangers of heat stress to exercising individuals on hot and humid days are well known. Recently, these dangers caused the deaths of some marines undergoing basic training. Every year, heat stress kills football players in pre-season training. Monitoring the individual and his environment can prevent such fatalities. Metabolic generation of heat should be considered not only a function of physical environment, but also a function of recent eating history (both amount and composition) sleeping patterns, and body composition.

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